REFINING THE ALGORITHM FOR A ZONAL CALCULATION OF HEAT TRANSFER IN AN OPEN-HEARTH FURNACE

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A refinement of the algorithm for calculating the parameters of radiative heat transfer has been developed and tested.

When studying the heat transfer in brightly lit open-hearth furnaces, it is necessary to take into account the nonuniform radiation characteristics of the flame and of the combustion products. The use of zonal calculation methods under these conditions meets with difficulties at the stage of determining the angular coefficients of interzonal radiation. The procedure for calculating the coefficient of radiative heat transfer between zones by statistical testing (Monte Carlo method) [1, 2] can be applied to a medium with a variable absorption coefficient under conditions of a complex geometry of the flame and the working space.

Basic to this procedure are the nonlinear algebraic equations of heat transfer and of heat balance between zones using the coefficients of interzonal radiation f_{ij} [2]. These coefficients represent the fraction of the energy radiated from zone i that is absorbed by zone j, taking into account multiple reflections at boundary surfaces. The calculation of radiative heat transfer by this method consists of two steps: first the coefficients f_{ij} are found by statistical tests, and then the system of nonlinear algebraic equations of the heat balance in the zones is solved, as a result of which the mean-zonal temperatures are determined. Calculation of the coefficients f_{ij} by statistical testing [1] involves the use of a digital computer to run a series of experiments on the observation of random radiation processes, and the transfer, absorption, and dispersion of the energy of individual radiation beams.

An experiment is considered to have been completed when, as a result of multiple absorptions in volume and surface zones, the energy of an individual radiation beam reaches a given sufficiently small density δ. Depending on the optical density of the medium occupying the radiative space and the absorptivity of the surface zones, this condition will be satisfied after some number of reflections of the individual beam from the boundary surfaces.

This method has been used to study the heat transfer in the working space of a steel-melting furnace under both bright flame and dark flame conditions [3]. The working space was simulated by a system of five computation segments corresponding to five respective charging orifices (Fig. 1a). Each computation segment consisted of two volume zones (the flame zone and the zone of recirculation of the combustion products) and three surface zones (walls, roof, and floor). The number and spacing of zones were chosen according to the particular construction of the steel-melting furnace, and also took into consideration the problems of determining the unknown temperature and thermal-flux distributions within the working space. The total number of zones was 25. In order to take into account the distribution of optical parameters of the flame more accurately, in calculating the radiation coefficients f_{ij} the computation segments were subdivided into smaller zones (Fig. 1b). The absorption factors K for the volume zones (Fig. 1b) were calculated from an experimental emissivity curve for the flame (Fig. 2) and the path length of the beam, equal to the flame width. An emissivity of 0.8 was assumed for the lining surface; a value of 0.6 was taken for the floor. A more thorough description of heat-transfer models is given in [3].

Table 1 lists the average numbers of reflections experienced by an individual radiation beam in the optical conditions and geometry of the working space of a melting furnace (Fig. 1), as a function of the final

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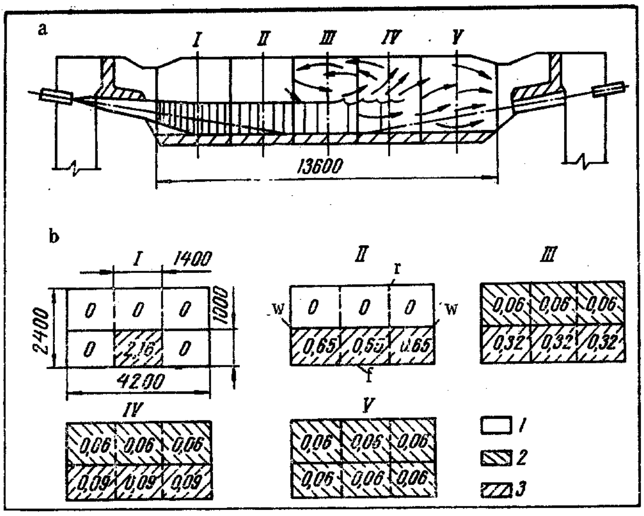

Fig. 1

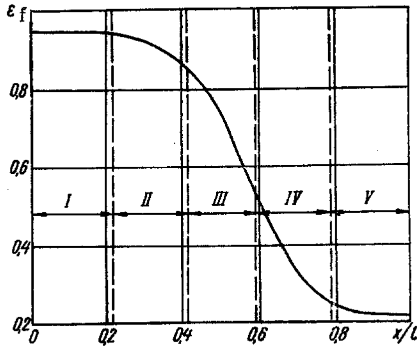

Fig. 2

Fig. 1. Schematic diagram representing the buildup of a flame through the working space in a steel-melting furnace (a), and the distribution of absorption factor K values over the volume zones (b): I-V) regions which correspond to charging orifices; the numbers shown in all volume zones indicate the value of K (1/m); r, w, f are the surface zones within the orifice regions: 1) transmitting layer; 2) absorbing layer; 3) flame.

Fig. 2. Variation of flame emissivity ε_f along the furnace length.

radiation flux density δ in an individual beam and of the flame emissivity (numerator) and also the machine time required on a Minsk-22 computer for the calculation of the coefficients f_{ij} using the zonal model of such a furnace (denominator). In the case of radiative systems containing media with a low optical density and with surface reflectivities close to unity, the number of reflections experienced by an individual beam is quite high. The computer time, which depends on the number of reflections, also becomes very long.

An analysis of this procedure and the work of other researchers [4-7] indicates the feasibility of determining the radiation coefficients f_{ij} in two steps. In the first step the generalized angular coefficients ψ_{ij} are determined by statistical testing. The second step is to calculate the f_{ij} coefficients, which take into account reradiation of energy from the boundary surfaces, by solving systems of linear algebraic equations.

Such a transition from generalized angular coefficients to resolvent angular coefficients with the aid of resolvent systems of linear algebraic equations was considered earlier by Yu. A. Surinov in [4, 5]. In [6] reradiation was taken into account by using the determinants of systems of linear algebraic equations which describe the radiative-energy balance for each surface. In [7] reradiation from the boundary surfaces was taken into account directly in solving the nonlinear algebraic equations of heat transfer and of zonal heat balance.

In view of all the above, in the method according to [2] it is proposed to determine the coefficients f_{ij} representing the fraction of the energy radiated from volume or surface zone i that is absorbed by surface zone j by solving the following system of linear algebraic equations

$$f_{ij} = \psi_{ij} A_j + \sum_{k=1}^{n} R_k \psi_{ik} f_{kj} \quad (i = 1, 2, \ldots, m+n). \tag{1}$$

The energy fraction f_{ij} absorbed by volume zone j is determined by solving the following system of equations:

$$f_{ij} = \psi_{ij} + \sum_{k=1}^{n} R_k \psi_{ik} f_{kj} \quad (i = 1, 2, \ldots, m+n). \tag{2}$$

TABLE 1. Average Number of Reflections Experienced by Individual Radiation Beams at the Boundary Surfaces (Numerator) and Machine Time (h) Necessary on Minsk-22 Computer to Calculate the Coefficients f_{ij} (Denominator)

Prescribed final-flux density	Appearance of flame	
	bright	dark
0,001	$\frac{4-5}{7,5}$	$\frac{5-6}{9}$
0,000001	$\frac{8-10}{15}$	$\frac{10-12}{18}$

Physically, these systems of equations signify that the total radiative energy received by zone j from radiating zone i is equal to the energy received by direct radiation from zone i to zone j plus the sum of radiation fluxes from zone i entering zone j after reflection from each surface zone k.

Such a refinement of the calculation algorithm makes it possible, by means of statistical testing, to terminate a single experiment whenever the given radiation beam strikes a boundary surface. The computer time for determining the generalized coefficients ψ_{ij} for this multizone model of a steel-melting furnace as a function of the final flux density δ and the flame emissivity is shorter by a factor of 4-12 (6-16 h) than that required to determine the coefficients f_{ij} by the Monte Carlo method and does not exceed 1.5 h (Table 1). The conversion to coefficients f_{ij} which take into account reradiation by solving systems of linear algebraic equations for the given case will require less than 2 min.

At the same time, the generality of the results is extended, since the generalized angular coefficients calculated by the Monte Carlo method may be used further to determine the coefficients f_{ij} which take into account reradiation in systems with surface zones of different absorption characteristics, and this simplifies the solution of problems of radiative heat transfer taking into account the spectral characteristics of the radiation from the surfaces. The smaller requirement for pseudorandom numbers in calculating the generalized coefficients improves the accuracy of results, since a high requirement on the amount of pseudorandom numbers is not always associated with a requirement for their uniform distribution on the interval (0, 1). When the zones are relatively small, furthermore, the use of the Monte Carlo method for the direct calculation of the coefficients f_{ij} will lead to a reduction in the accuracy, since the number of individual radiation beams traced, both incident and consequently also reflected from a surface zone is considerably less than the number of individual radiation beams originally emanating from the radiating zone.

An analysis of Eqs.(1) and (2) shows that it is convenient to begin the calculations by determining the radiation coefficients f_{ij} for the surface zones. This allows the solution to be limited to an n-th order system of equations, where n is the number of surface zones. The second step in solving the given problem (determination of the coefficients f_{ij} for the volume zones) reduces, as a result of this, to a simple calculation according to Eqs.(1) and (2), their right-hand sides containing no unknowns.

Systems (1) and (2) ($i = 1, 2, \ldots, n$) will be represented in matrix form (the number of systems of equations is equal to $m + n$, i.e., the number of absorbing zones j):

$$A_{ik} f_{ij} = b_{ij}, \tag{3}$$

where

$$A_{ik} = \begin{cases} -R_k \psi_{ik} & \text{for} \quad i \neq k, \\ 1 - R_k \psi_{ik} & \text{for} \quad i = k \end{cases} \tag{4}$$

and

$$\begin{aligned} b_{ij} &= \psi_{ij} A_j, \text{ if j is a surface zone;} \\ b_{ij} &= \psi_{ij}, \quad \text{if j is a volume zone.} \end{aligned} \tag{5}$$

Examining Eqs.(1)-(4), we find that matrix A_{ik} ($n \times n$ dimensional) is the same for n systems of Eqs.(1) as for m systems of Eqs.(2). The difference between these systems of equations consists only in the different magnitude of vectors b_{ij}, which can be expressed in terms of a maxtrix of dimension $n \times (m + n)$. A survey of the methods by which systems of linear equations are solved [8, 9] shows that solution by means of the inverse matrix is most suitable in this case, since, having transformed matrix A_{ik} (4) of the coefficients in systems (1) and (2) into the inverse matrix A_{ik}^{-1}, the solution of each individual system of n equations can be obtained by multiplying the inverse matrix A_{ik}^{-1} (which is common to all systems of equations) by the vector b_{ij} which corresponds to the system of equations in question i.e.,

$$f_{kj} = A_{ik}^{-1} b_{ij}. \tag{6}$$

TABLE 2. Values of the Radiation Coefficients f_{ij} Calculated by the Proposed Algorithm (I) and Directly by the Monte Carlo Method (II). Radiation from the flame (Volume Zone No. 7)*

Absorbing zone		No. of computation segment	f_{ij} (I)	f_{ij} (II)
Volume zones	1	I	0,001	0,001
	2		0,000	0,000
	3	II	0,006	0,007
	4		0,000	0,000
	5	III	0,044	0,043
	6		0,012	0,012
	7	IV	0,090	0,087
	8		0,034	0,035
	9	V	0,015	0,017
	10		0,017	0,015
Surface zones	11	I	0,002	0,002
	12		0,003	0,003
	13		0,000	0,000
	14	II	0,004	0,005
	15		0,007	0,008
	16		0,001	0,001
	17	III	0,041	0,040
	18		0,048	0,047
	19		0,021	0,022
	20	IV	0,203	0,210
	21		0,116	0,115
	22		0,176	0,176
	23	V	0,063	0,065
	24		0,061	0,056
	25		0,035	0,030

*The zone numbers are assigned as follows: 1, 3, 5, 7, 9 are flame zones; 2, 4, 6, 8, 10 are zones of the radiation-transmitting and of the radiation-absorbing layer; 11, 14, 17, 20, 23, are wall zones; 12, 15, 18, 21, 24 roof zones; 13, 16, 19, 22, 25 floor zones.

Therefore, although matrix inversion requires more computation time than other procedures for solving systems of linear equations, the total computation time for solving m + n systems of equations is short, because the most laborious operation, matrix inversion, must be performed only once.

The results obtained by calculating the radiation coefficients f_{ij} for the zonal model of a steel-melting furnace (Fig. 1) according to the procedure of [1] before and after refinement indicate that the respective values do not differ much, even though the calculation schedule has been changed considerably. The absolute divergence does not exceed 0.023 and in individual rows (Table 2) it does not exceed 0.007. Thus, the accuracy of determining the radiation coefficients f_{ij} depends essentially on the accuracy of the Monte Carlo method in calculating the angular coefficients. Calculations performed in [1, 10], as well as our evaluation of the error in determining geometrical and generalized angular coefficients for the different faces of a parallelepiped or a cube show that for 2000 statistical tests an accuracy of 2-2.5% is attained. The accuracy can be improved by increasing the number of tests and by matching this with a program for generating pseudorandom numbers.

NOTATION

K is the volume absorption factor for the medium, m^{-1};
A_j is the absorptivity (emissivity) of surface zone j;
R_k is the reflectivity of surface zone k;
f_{ij} is the coefficient of the radiation from zone i to zone j, taking into account reradiation from surface zones;
ψ_{ij} is the mean generalized angular coefficient of radiation from zone i to zone j;
δ is the prescribed final flux density in a radiation beam;
ε_f is the emissivity of the flame;
l is the length of the working space in the furnace, m;

A_{ik} is the matrix of the coefficients in the system of linear algebraic equations;
A_{ik}^{-1} is the inverse matrix of A_{ik}.

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